Intelligent Robot Control

Lecture 8: Advances in phase-state-systems

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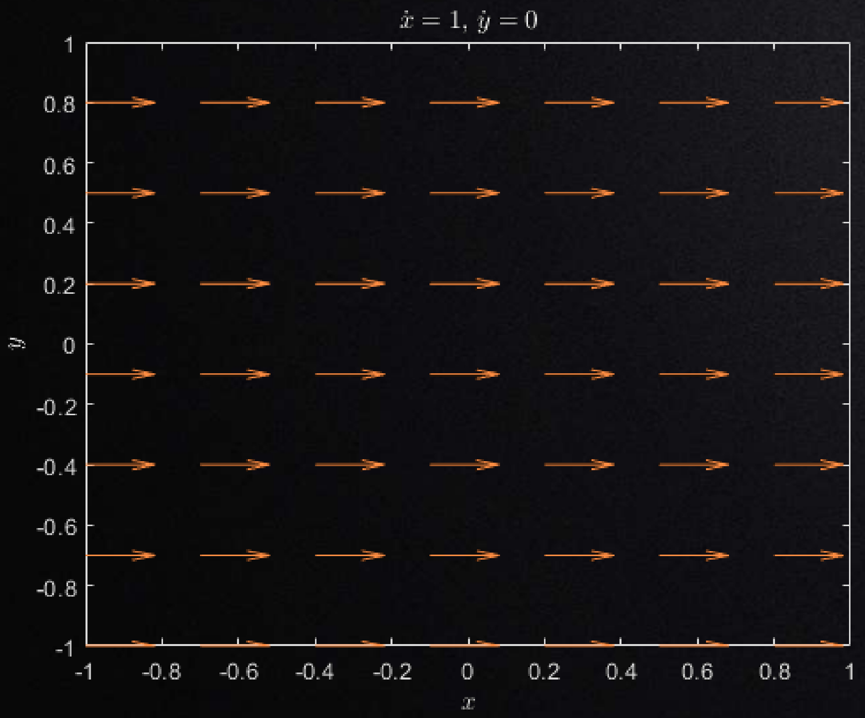
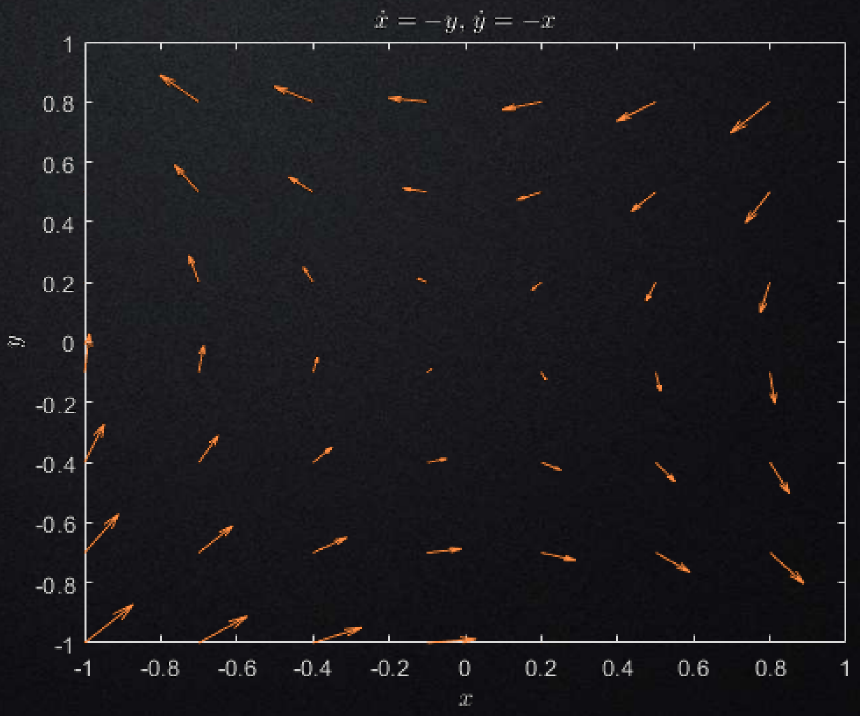
2D dynamical system

- Aka.: planar system
  - $\dot{x} = f(x, y)$
  - $\dot{y} = g(x, y),$
- f and g are the vector fields that describes the evolution of the 2D state variables x(t) and y(t)
- Examples:





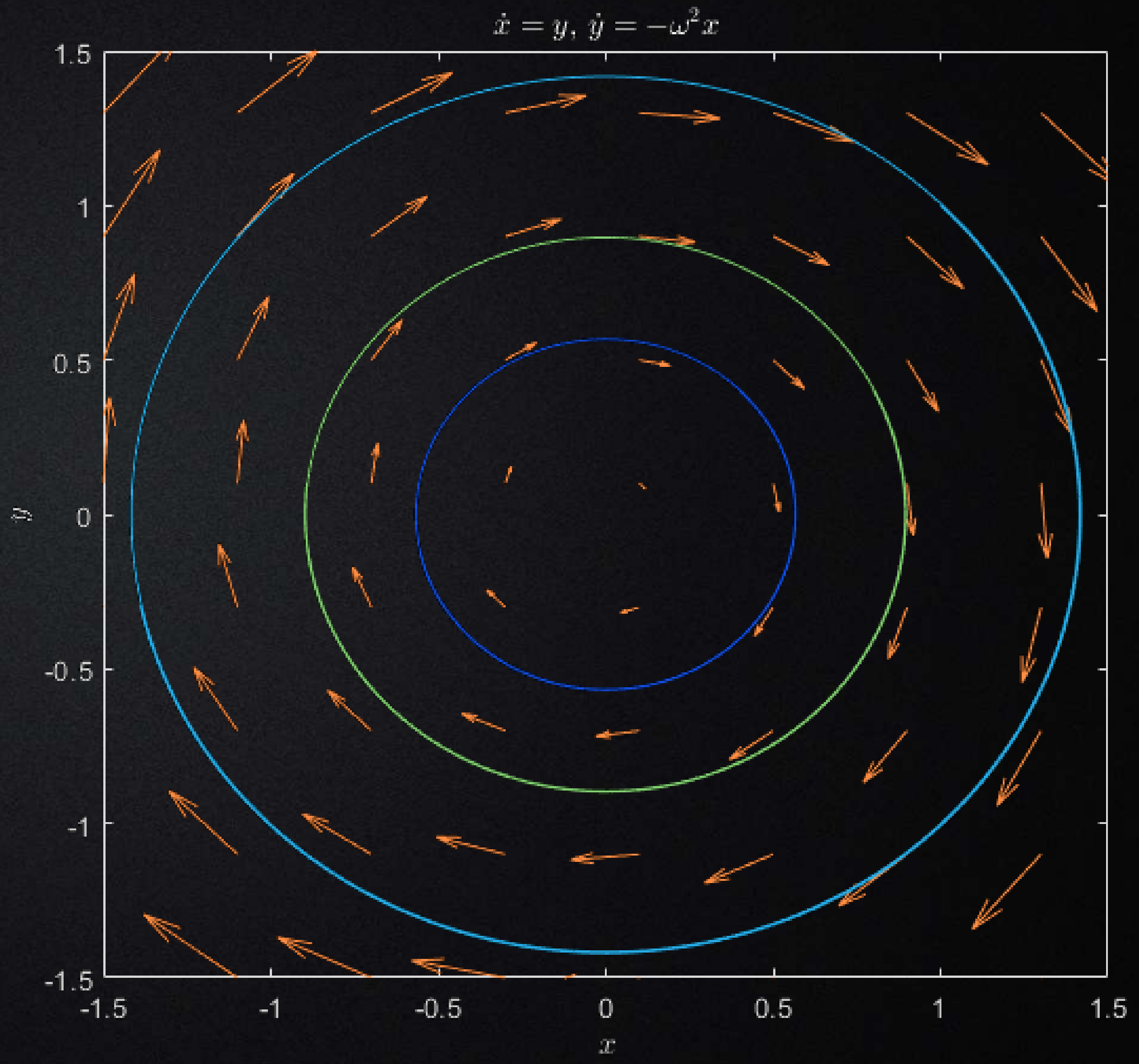
Note: a **nullcline** is a curve in the phase plane on which the vector field is defined by the differential equation points in a particular direction.

We can get oscillations in 2D

Center

- Can be observed in linear systems

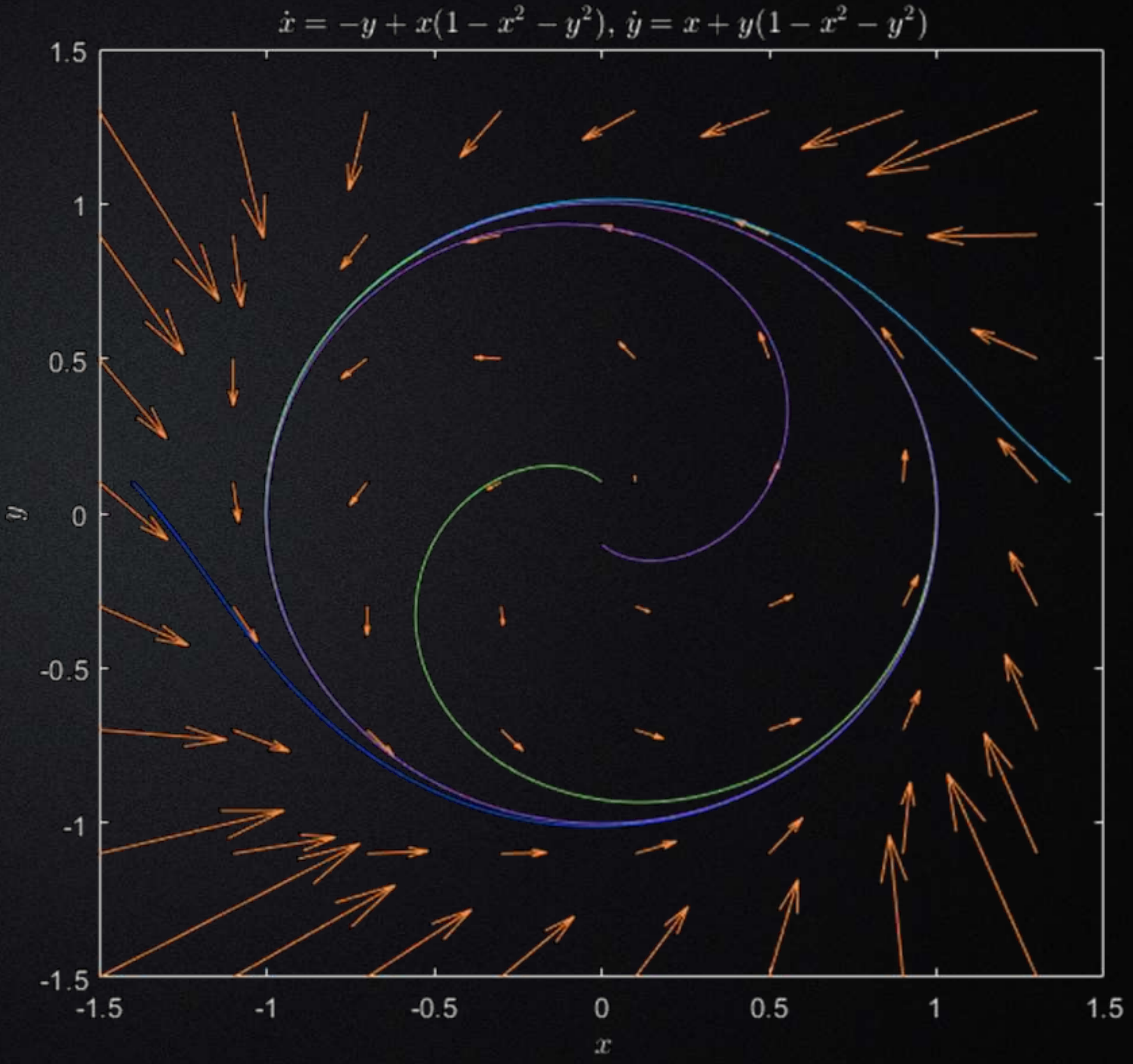
- The amplitude of oscillation typically depend on the initial conditions.



We can get oscillations in 2D

Limit cycles

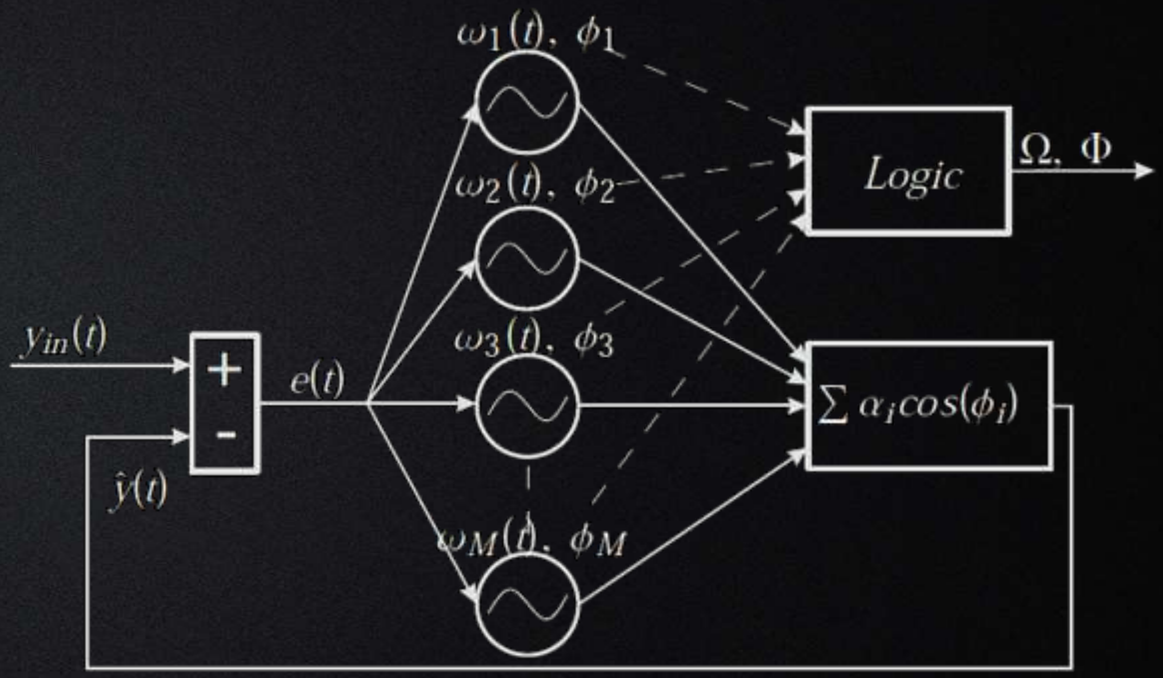
- Isolated periodic orbits in the phase space
- Are inherently a nonlinear phenomenon
- Usually is difficult to guess the existence of the limit cycles only by looking at the set of ODEs



Pool of oscillators

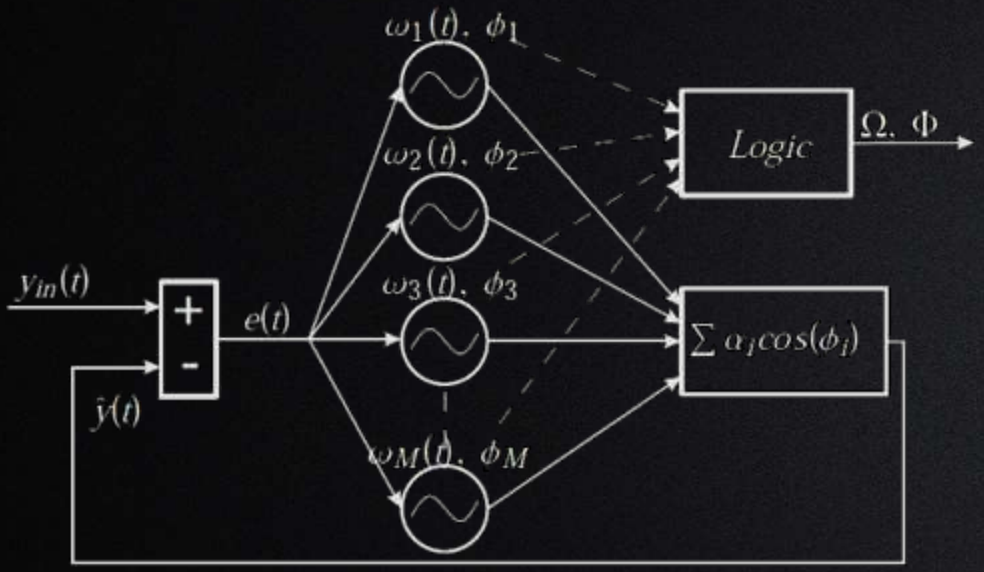
(original approach)

- Extracts the frequency and phase of the input signal
- Based on a pool of adaptive frequency oscillators
- Feedback loop allows extraction of several frequency components
- No additional signal transformations are needed (e.g. FFT)
- Significant drawback: requires a logic algorithm



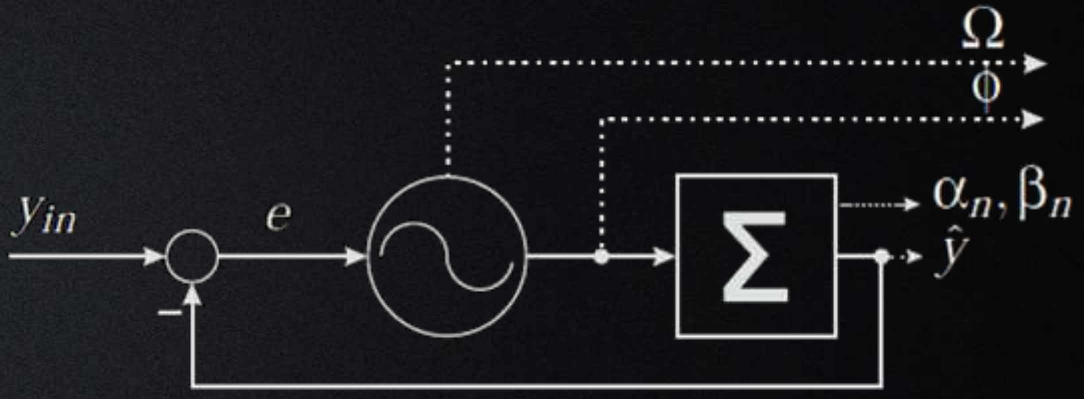
Redesign the pool of oscillators

Pool of AFOs:



$$\dot{\phi}_i = \omega_i - Ke\sin(\phi_i),$$
$$\dot{\omega}_i = -Ke\sin(\phi_i),$$

AFO with adaptive Fourier series:

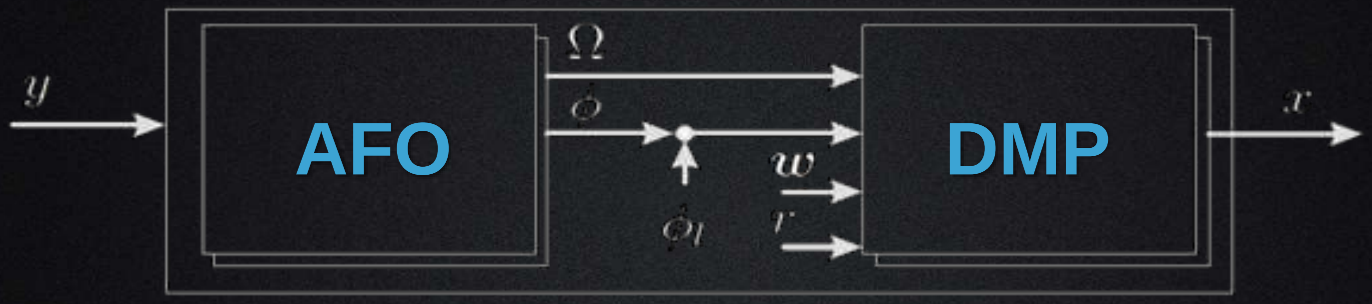


$$\dot{\phi} = \Omega - Ke\sin(\phi),$$
$$\dot{\Omega} = -Ke\sin(\phi),$$

Petrič T., Gams A., Ijspeert A., Žlajpah L., On-line frequency adaptation and movement imitation for rhythmic robotic tasks, *The international journal of robotics research, 2011.*

Towards robot control using adaptive limit cycles

- Basic idea: approaches that combine both frequency extraction and waveform learning:



- Original two-layered imitation system [1]:
  - Extracting the frequency
  - Adaptation to input signal

  } (AFO)

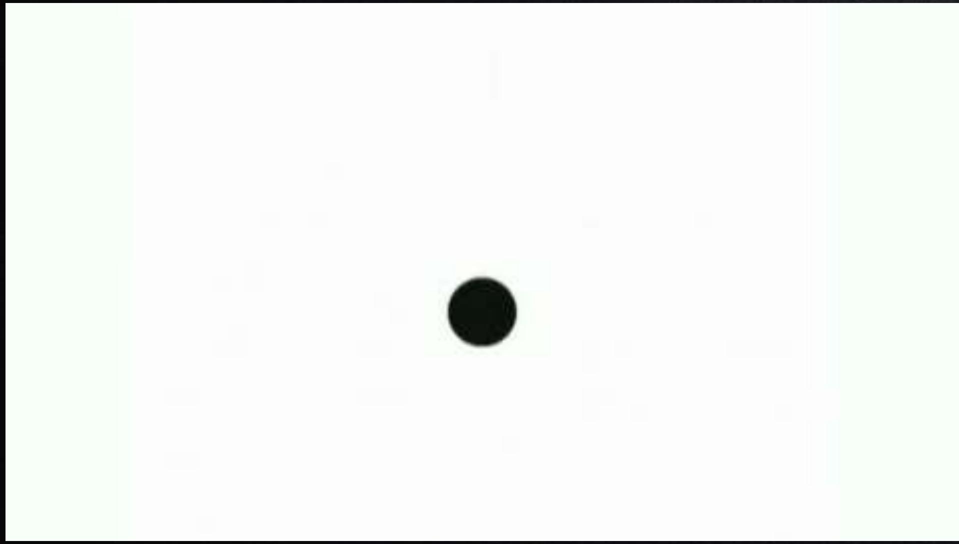
  - Learning the waveform
  - Modulating the waveform

  } (DMP)

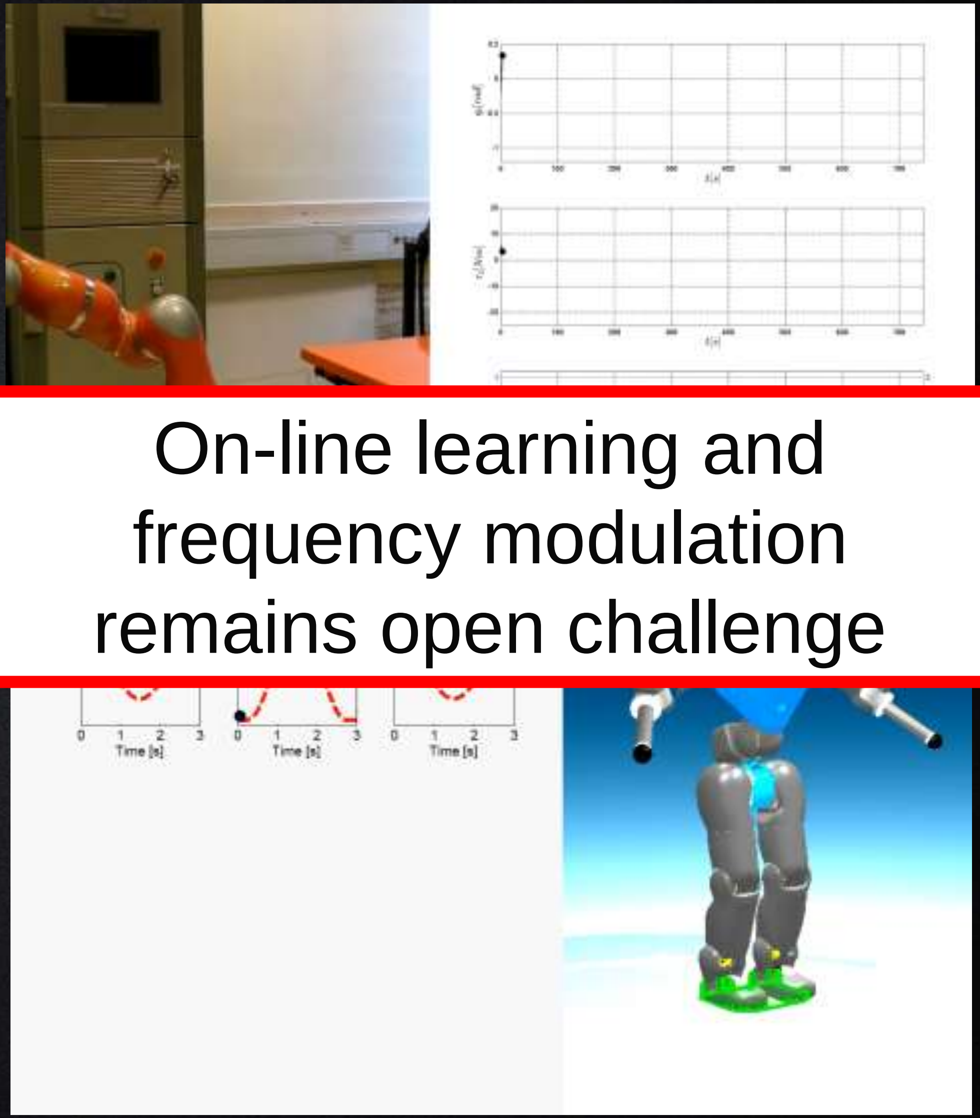
1. Gams A., Ijspeert A., Schaal S., Lenarčič J., On-line learning and modulation of periodic movements with nonlinear dynamical systems, *Autonomous robots*

AFO + (DMP || CMP)

- DMP



- CMP



On-line learning and frequency modulation remains open challenge

- **T. Petric**, A. Gams, L. Colasanto, A. J. Ijspeert, and A. Ude, "Accelerated Sensorimotor Learning of Compliant Movement Primitives" *IEEE Transactions on Robotics*, 2018
- Denisa, A. Gams, A. Ude, and **T. Petric**, "Learning Compliant Movement Primitives Through Demonstration and Statistical Generalization" *IEEE/ASME Trans. Mechatronics*, 2016.

Phase-synchronization

- Original AFS

$$\dot{\phi} = \Omega - Ke\sin(\phi),$$
$$\dot{\Omega} = -Ke\sin(\phi),$$

$$e = y_{in} - \hat{y},$$

$$\hat{y} = \alpha_0 + \sum_{i=1}^{M} (\alpha_i \cos(i\phi) + \beta_i \sin(i\phi)),$$

$$\dot{\alpha}_o = \eta e,$$
$$\dot{\alpha}_i = \eta \cos(i\phi) e,$$
$$\dot{\beta}_i = \eta \sin(i\phi) e,$$

- AFS – with phase synchronization

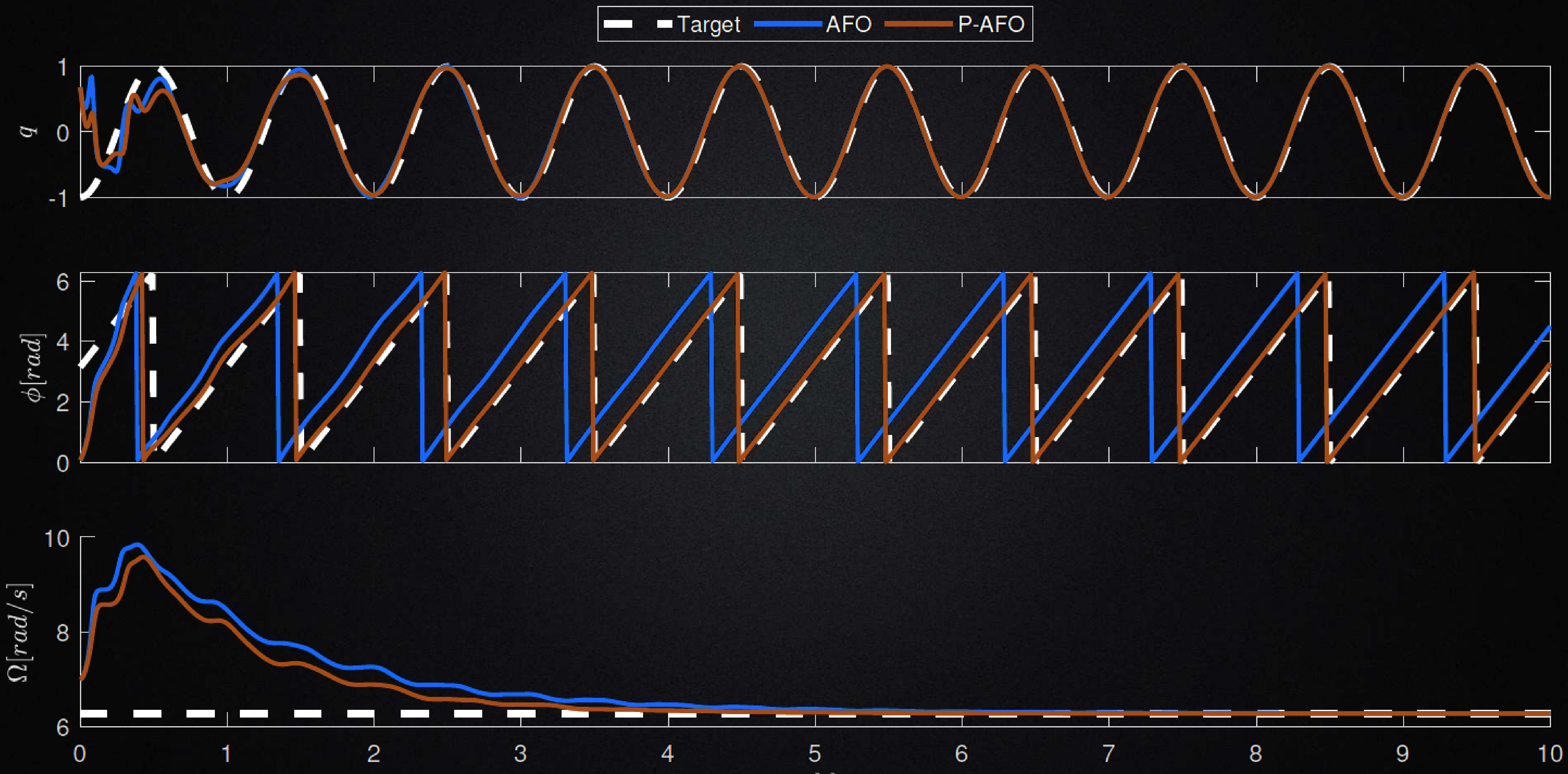
$$\dot{\phi} = \Omega - Ke\sin(\phi),$$
$$\dot{\Omega} = -Ke\sin(\phi),$$

$$e = y_{in} - \hat{y},$$

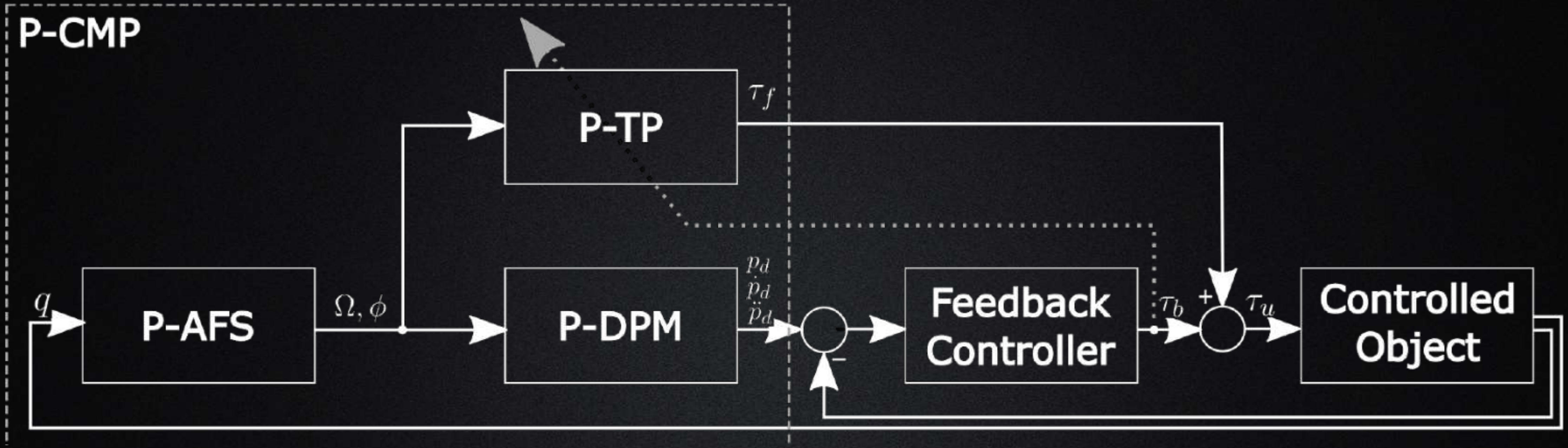
$$\hat{y} = \alpha_0 + \sum_{i=1}^{M} (\alpha_i \cos(i\phi)) + \sum_{j=2}^{M} (\beta_j \sin(j\phi)),$$

$$\dot{\alpha}_o = \eta e,$$
$$\dot{\alpha}_i = \eta \cos(i\phi) e,$$
$$\dot{\beta}_j = \eta \sin(j\phi) e,$$

Convergence of an AFO and P-AFO systems



Robots - torque control



- P-AFS – adaptive phase oscillator
- P-DMP – periodic dynamic movement primitive
- P-TP – periodic torque primitive

P-DMP - Dynamic Movement Primitives

- Dynamic Movement Primitives = nonlinear system of differential equations
- Periodic case (Ijspeert, Nakanishi & Schaal, 2002):

Dynamic Movement Primitives

$$\dot{z} = \Omega\left(\alpha_z\left(\beta_z\left(g-y\right)-z\right)+\frac{\sum_{i=1}^{N}\Psi_i w_i r}{\sum_{i=1}^{N}\Psi_i}\right),$$

$$\dot{y} = \Omega z$$

Kernel functions

$$\Psi_i = \exp\left(h\left(\cos(\phi - c_i)-1\right)\right),$$

Already frequency and phase dependent

- DMPs are not explicitly dependent on time.
- DMPs can easily be modulated to adapt to different conditions

P-TP – Torque primitive

- Corresponding torques are encoded as a linear combination of basis functions

- Original TP

$$\tau_f(\phi) = \frac{\sum_{i=1}^{N} \Psi_i w_i}{\sum_{i=1}^{N} \Psi_i}$$

Kernel functions

$$\Psi_i = \exp\left(h\left(\cos(\phi - c_i) - 1\right)\right),$$

Target function

$$e_r(t) = f_{\mathrm{tar}}(\mathrm{t}) - \mathrm{w}_\mathrm{i}^\mathrm{t}\mathrm{r}(\mathrm{t})$$

$$f_{\mathrm{tar}}(\mathrm{t}) = \tau(\mathrm{t})$$

Regression

$$w_i^{t+1} = w_i^t + \Psi_i P_i^{t+1} r(t) e_r(t),$$

- P-TP – periodic torque primitive

$$\tau_f(\Omega, \phi) = \frac{\sum_{i=1}^{N} \sum_{j=1}^{K} v_{i,j} \psi_i(\phi) \Psi_j(\Omega)}{\sum_{i=1}^{N} \sum_{j=1}^{K} \psi_i(\phi) \Psi_j(\Omega)}$$

Kernel functions

$$\psi_i(\phi) = \exp\left(h^{\phi}\left(\cos\left(\phi - c_i^{\phi}\right) - 1\right)\right)$$

$$\Psi_j(\Omega) = \exp\left(-h^{\Omega}\left(\Omega - c_i^{\Omega}\right)^2\right)$$

Feedback error learning

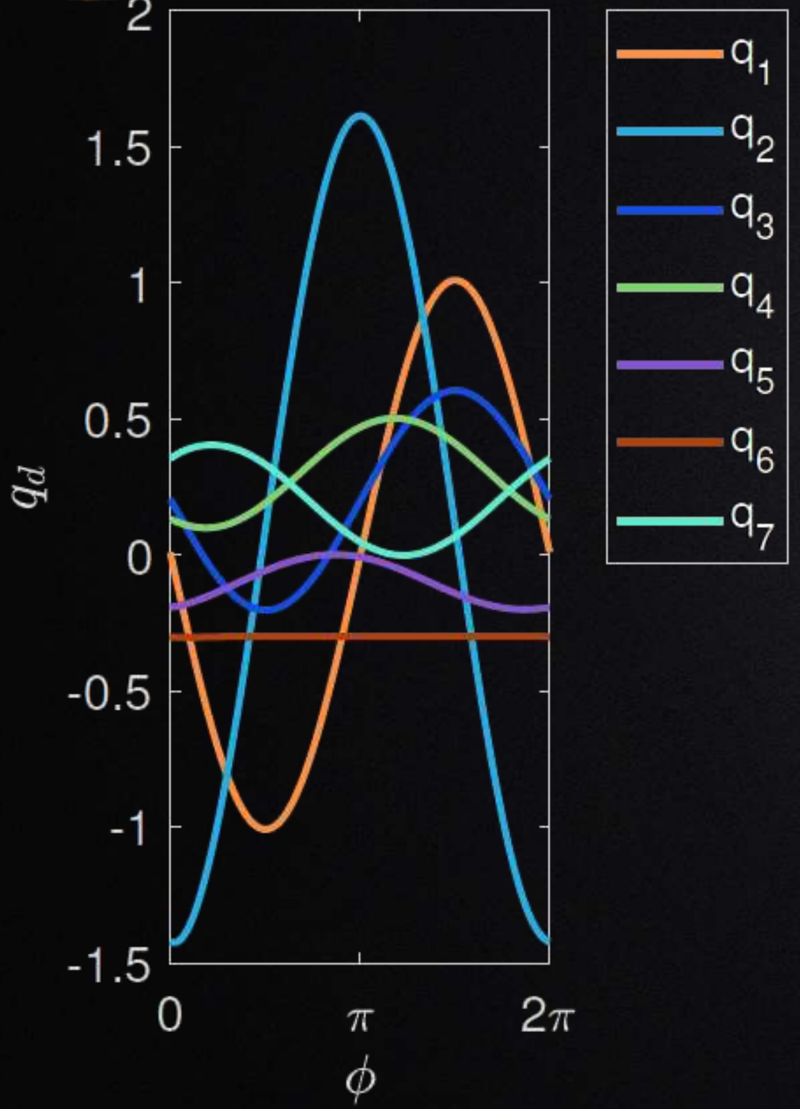
$$\tau_u = \tau_b + \tau_f$$

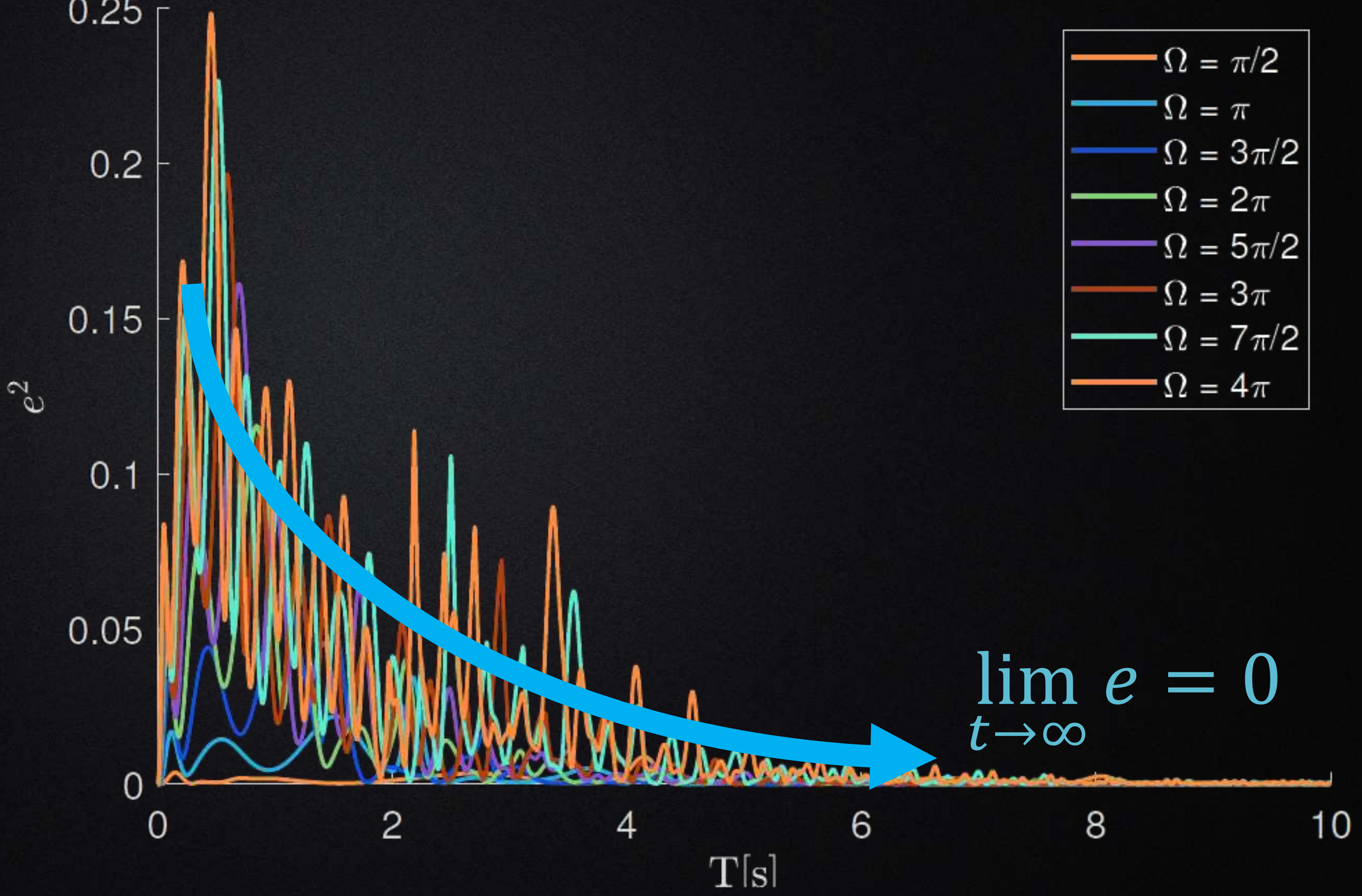
$$\tau_b = K_p e + K_d \dot{e} + K_i \ddot{e}$$

$$\dot{v}_{i,j} = \mathrm{K}_v \tau_b$$

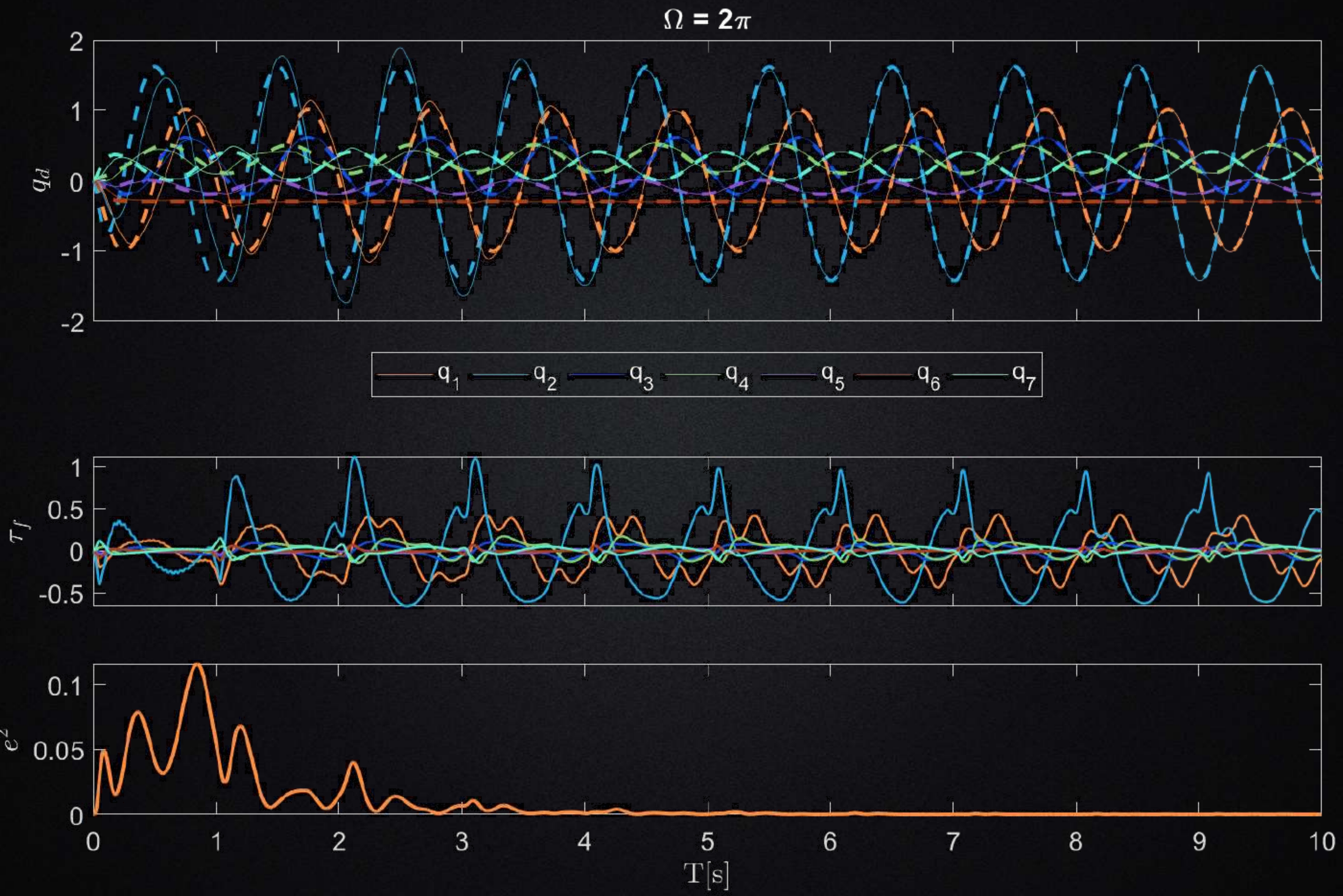
Frequency and phase dependent

Learning of internal dynamical models

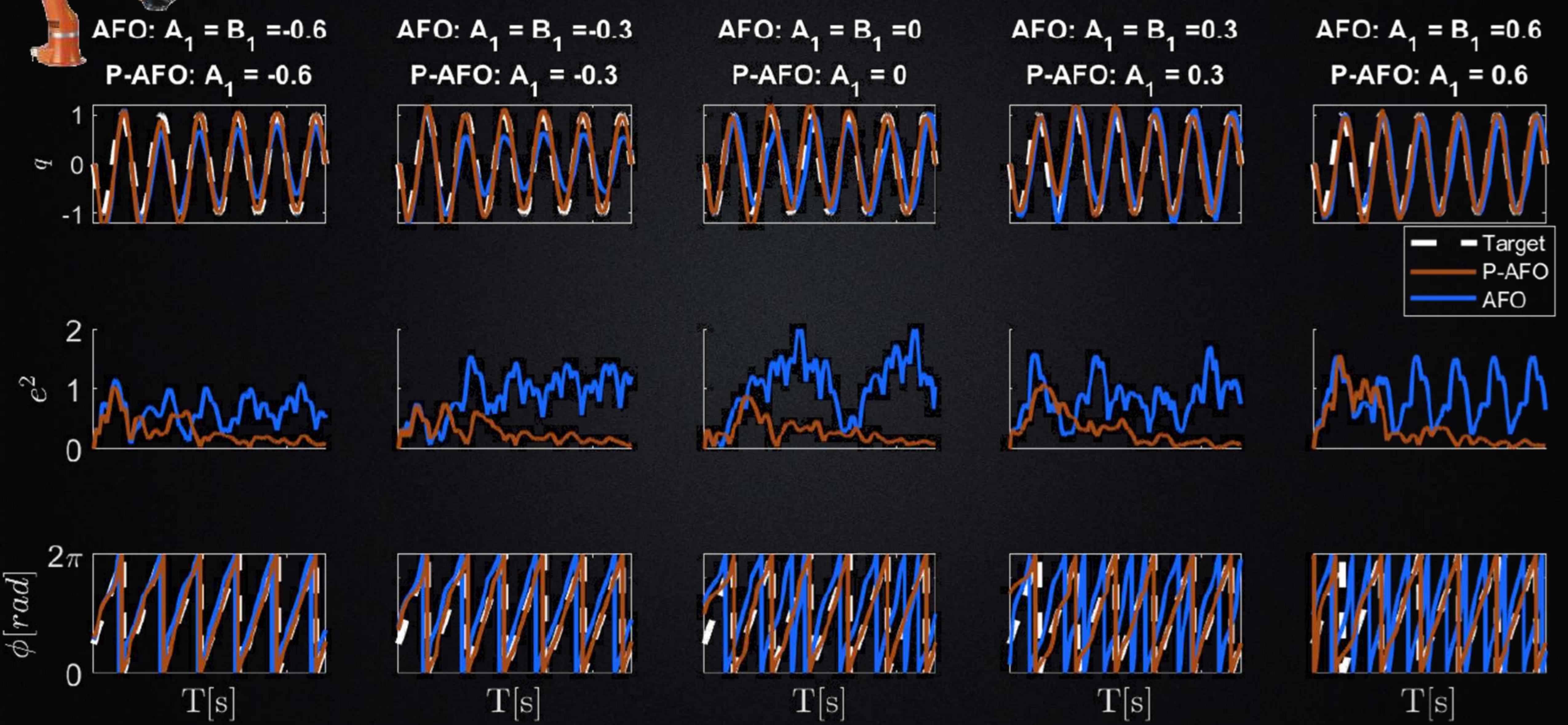




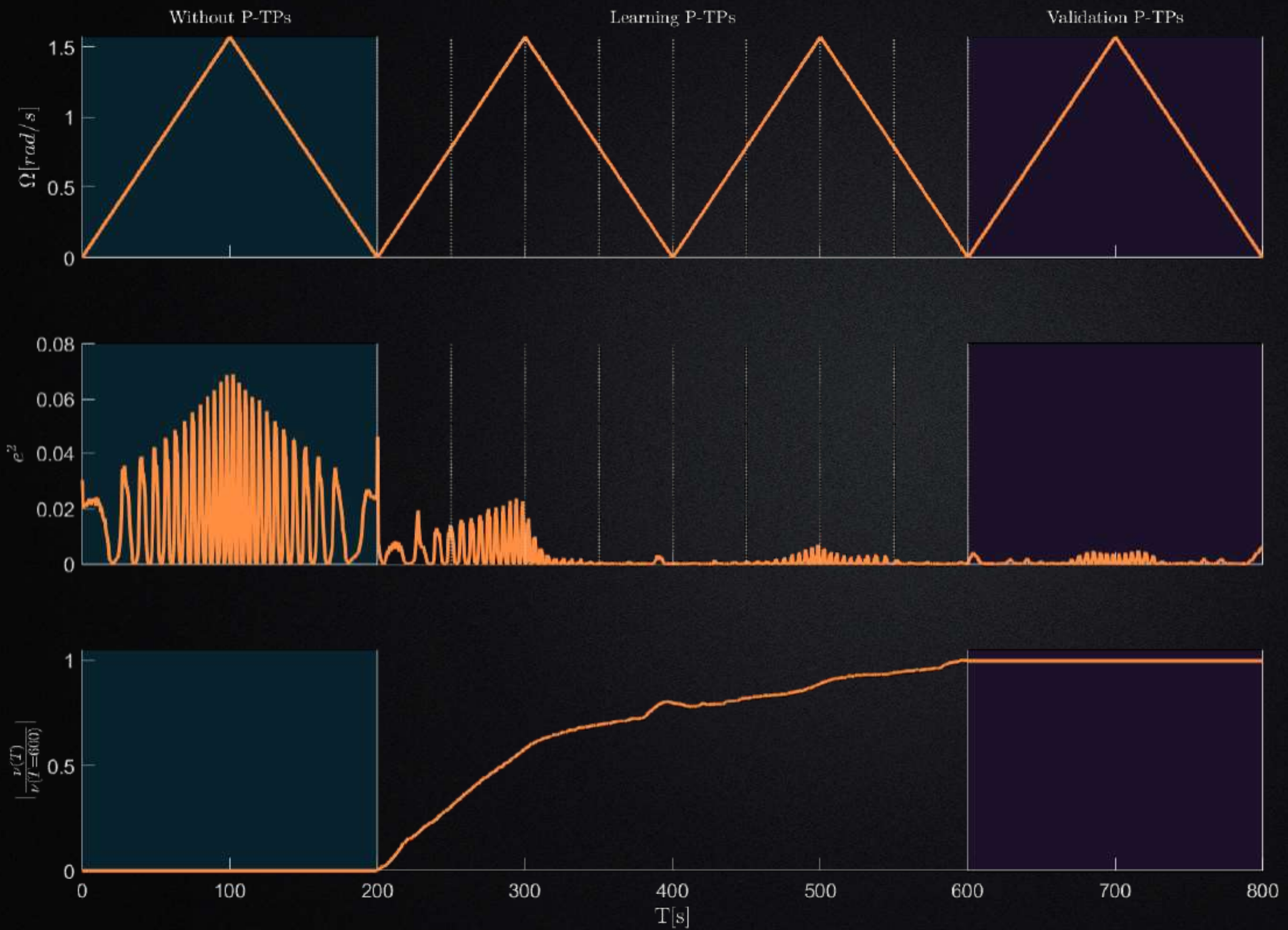
Learning of internal dynamical models

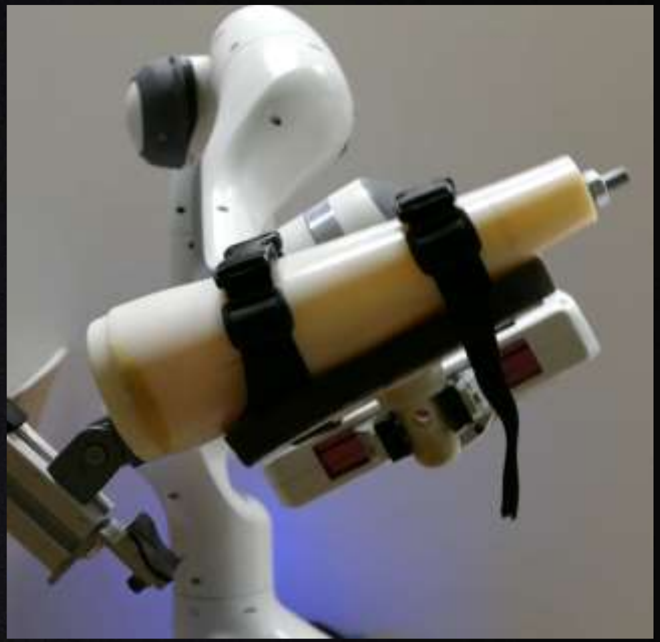


Learning of internal dynamical models

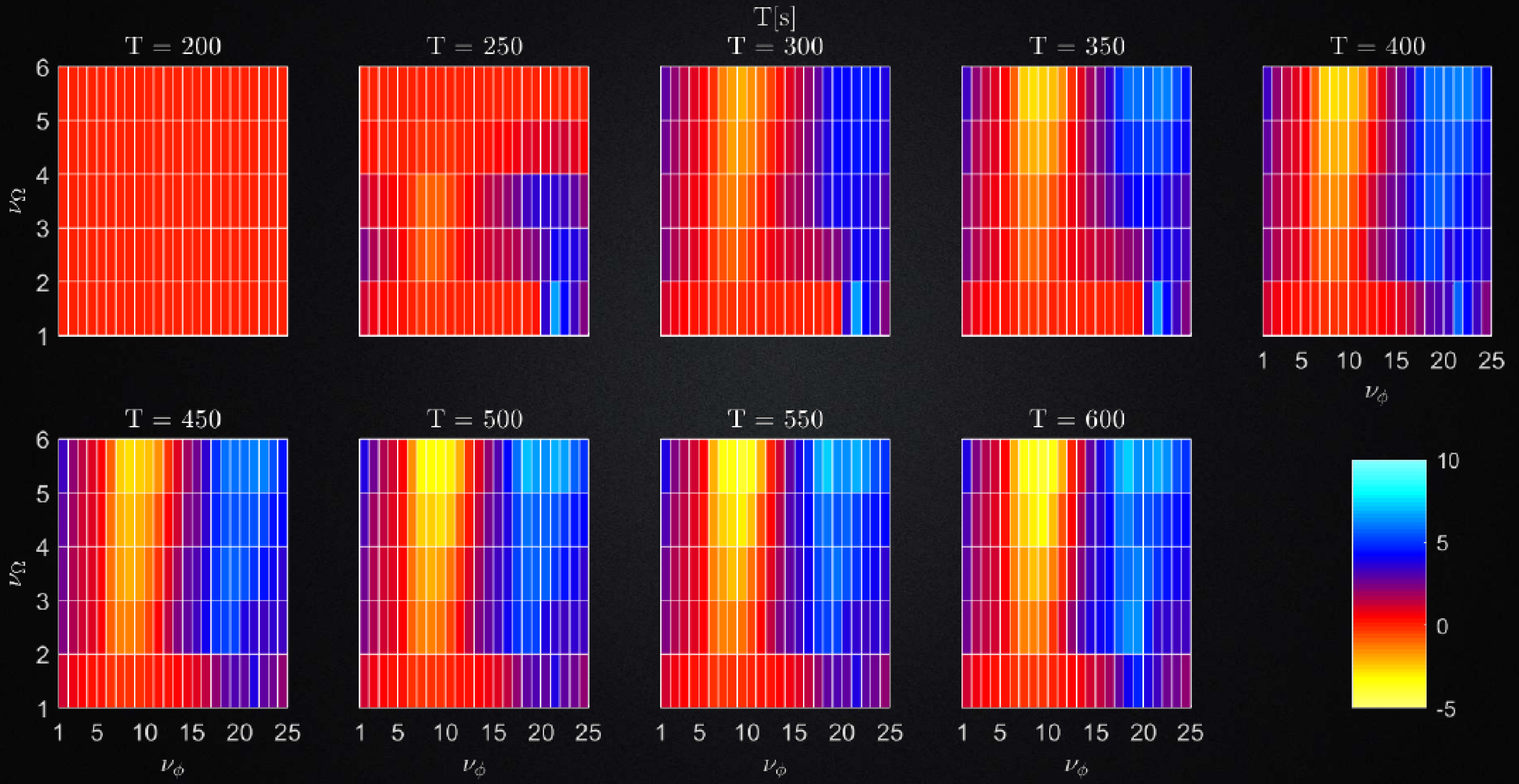


Physically simulated human elbow stretching tasks





P-TPs weight matrix values



ORIGINAL RESEARCH article
Front. Neurorobot., 12 November 2020 | https://doi.org/10.3389/fnbot.2020.599889

Check for updates

Phase-Synchronized Learning of Periodic Compliant Movement Primitives (P-CMPs)

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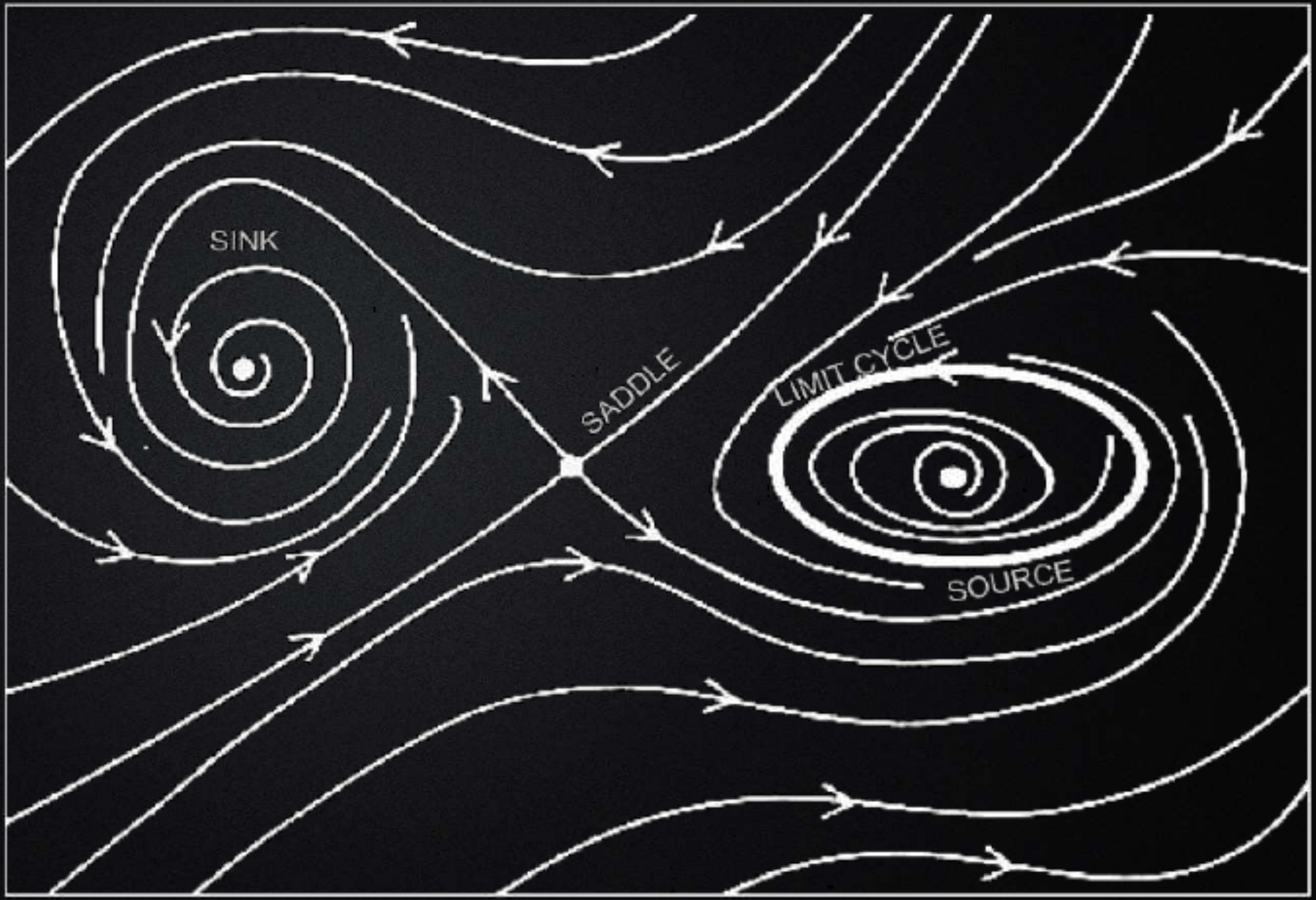
Autonomous trajectory and torque profile synthesis through modulation and generalization require a database of motion with accompanying dynamics, which is typically difficult and time-consuming to obtain. Inspired by adaptive control strategies, this paper presents a novel method for learning and synthesizing Periodic Compliant Movement Primitives (P-CMPs). P-CMPs combine periodic trajectories encoded as Periodic Dynamic Movement Primitives (P-DMPs) with accompanying task-specific Periodic Torque Primitives (P-TPs). The state-of-the-art approach requires to learn TPs for each variation of the task, e.g., modulation of frequency. Comparatively, in this paper, we propose a novel P-TPs framework, which is both frequency and phase-dependent. Thereby, the executed P-CMPs can be easily modulated, and consequently, the learning rate can be improved. Moreover, both the kinematic and the dynamic profiles are parameterized, thus enabling the representation of skills using corresponding parameters. The proposed framework was evaluated on two robot systems, i.e., Kuka LWR-4 and Franka Emika Panda. The evaluation of the proposed approach on a Kuka LWR-4 robot performing a swinging motion and on Franka Emika Panda performing an exercise for elbow rehabilitation shows fast P-CTPs acquisition and accurate and compliant motion in real-world scenarios.

TOWARDS PHASE-STATE SYSTEM

Phase portrait for 2D system

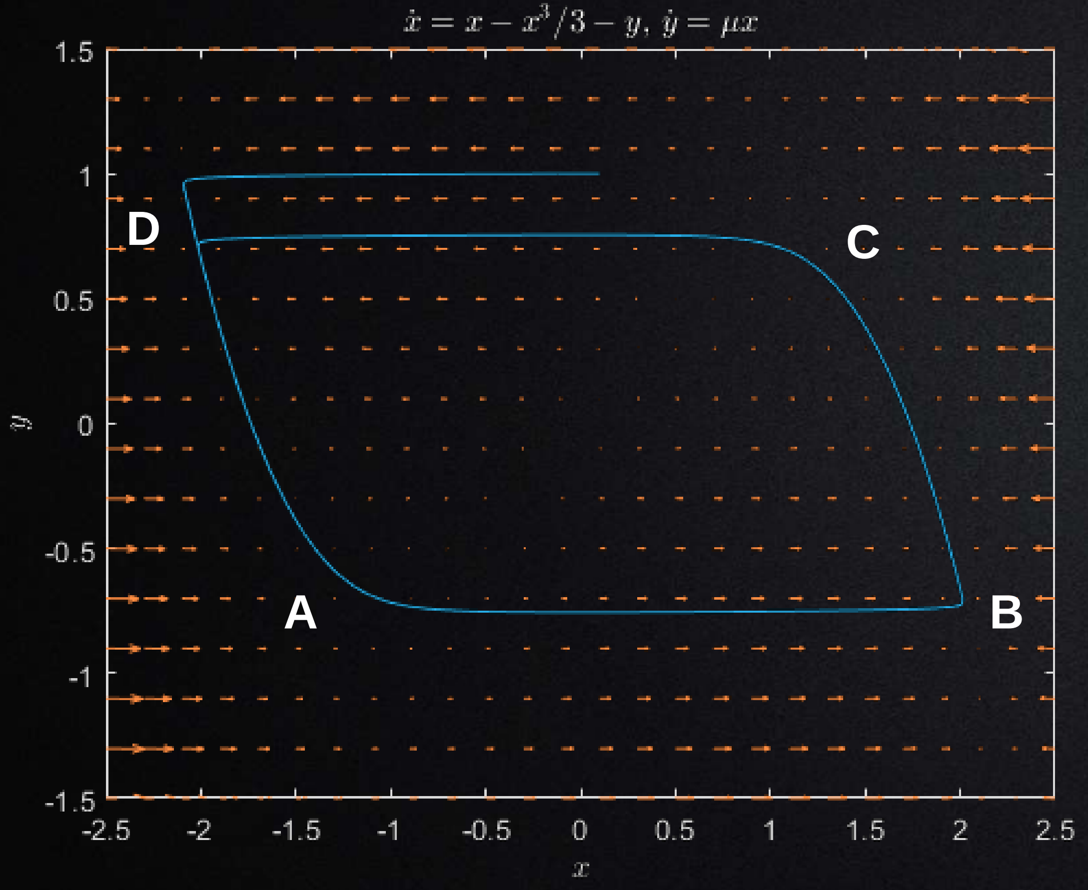
- Trajectories in phase portrait does not cross each other

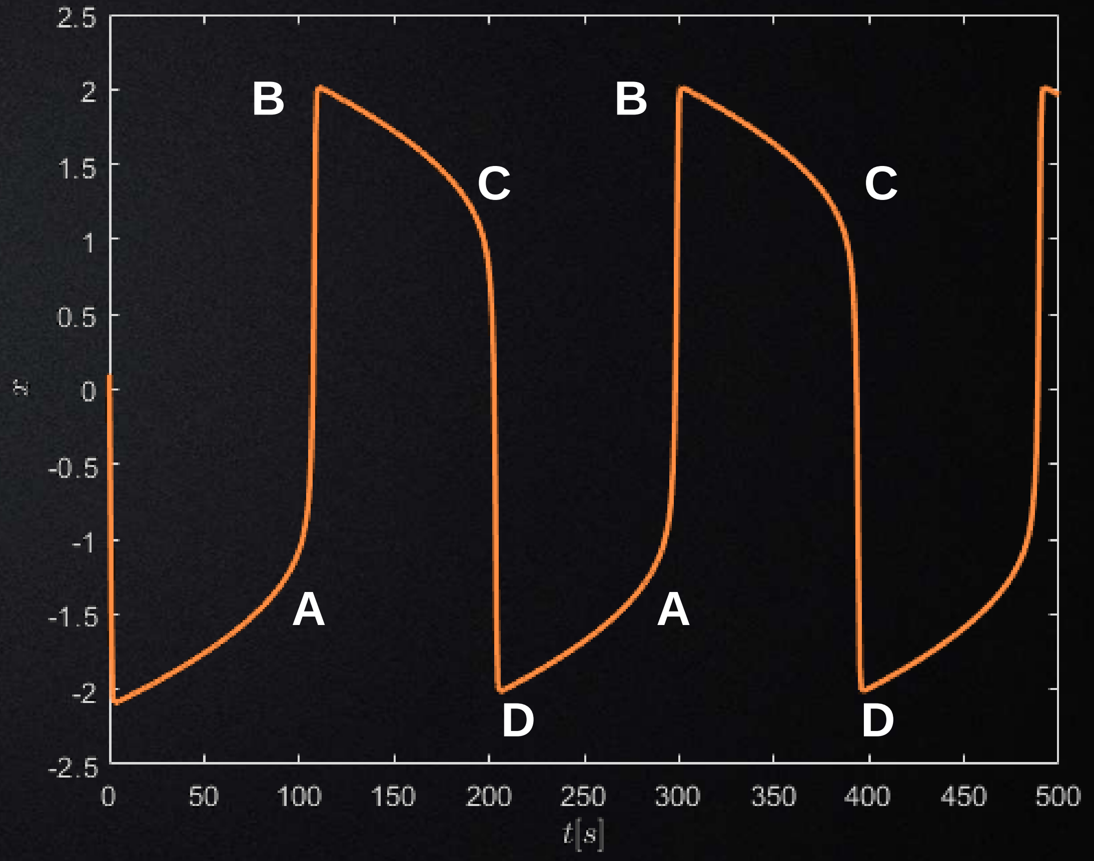
- Consequence of existence and uniqueness property of solution



Relaxation oscillators

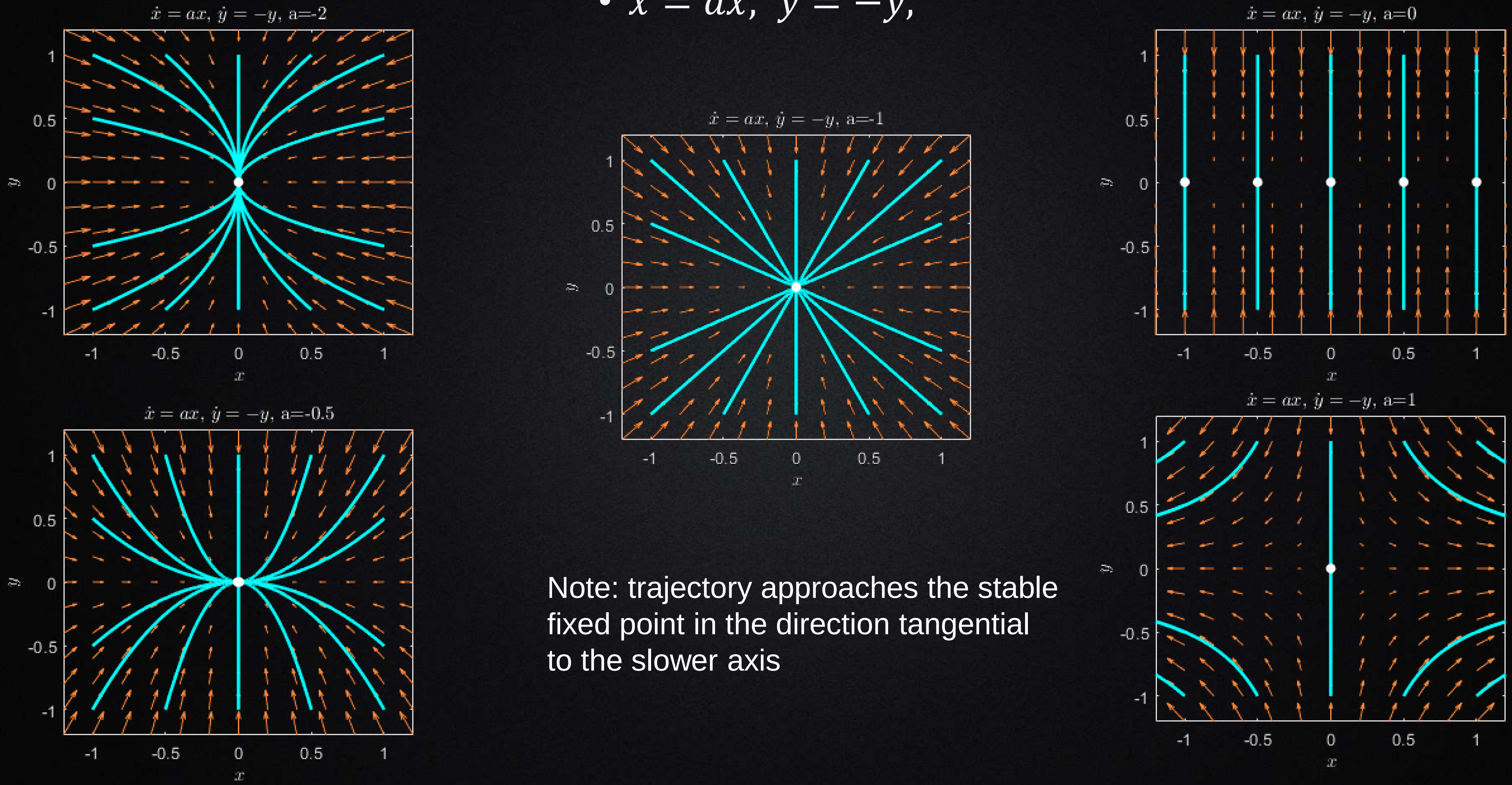
- Exhibits fast and slow time scale





2D uncoupled system - bifurcation

- $\dot{x} = ax,\ \dot{y} = -y,$

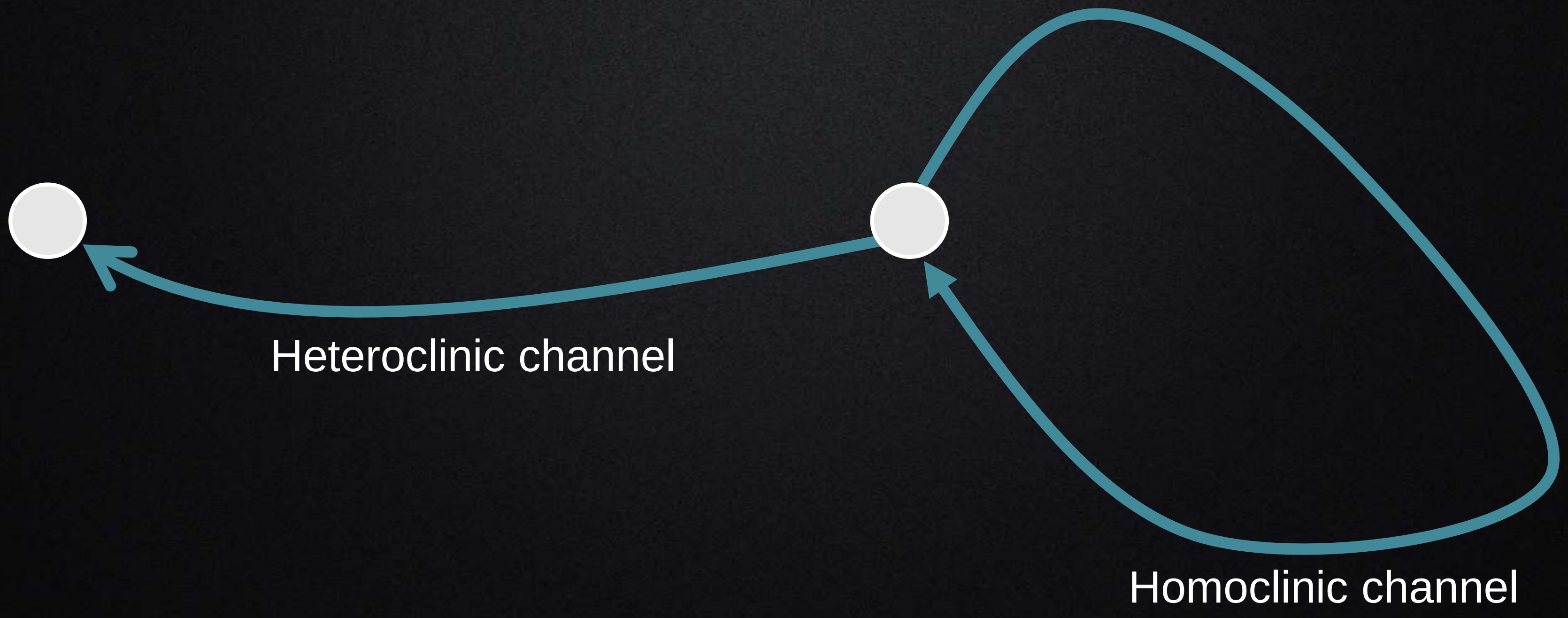


Note: trajectory approaches the stable fixed point in the direction tangential to the slower axis

Homoclinic and heteroclinic trajectories

- A trajectory is homoclinic if it originates from the terminates at the same equilibrium point

- A trajectory is heteroclinic if it originates at one equilibrium and terminates at the different equilibrium point

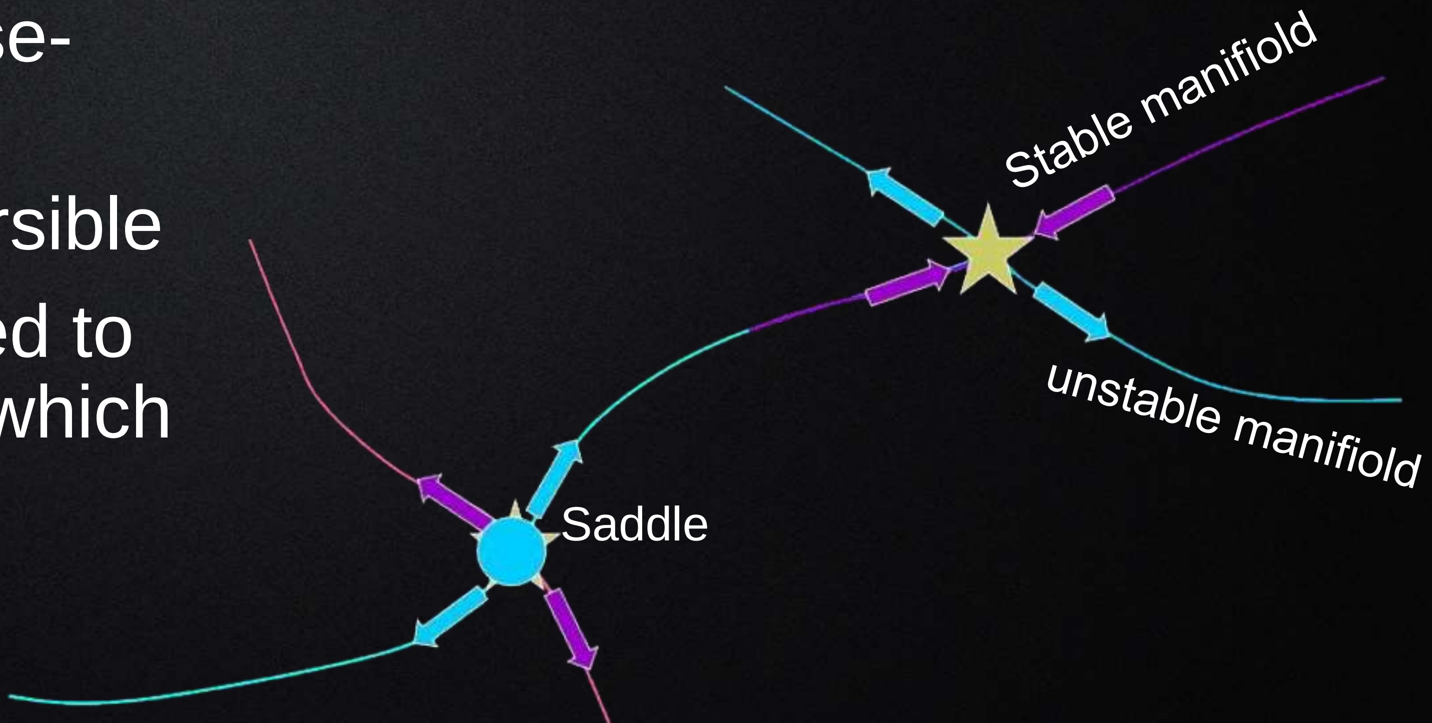


Towards dynamical system for continuous and reactive behaviors

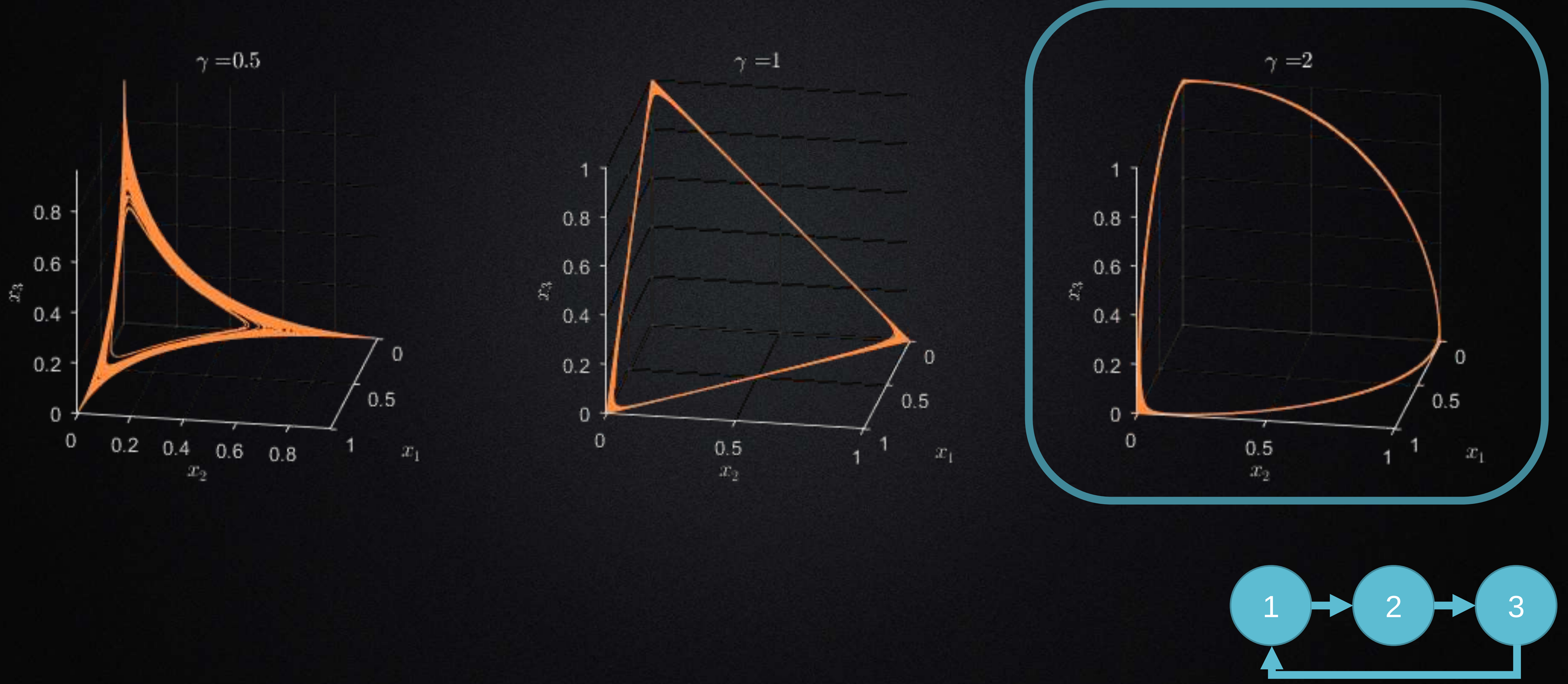
- Network of Stable Heteroclinic Channels (SHC)

$$\dot{x} = x \circ (\alpha - P \cdot x^{\gamma}) \cdot \eta(t) + \dot{\delta}(t)$$

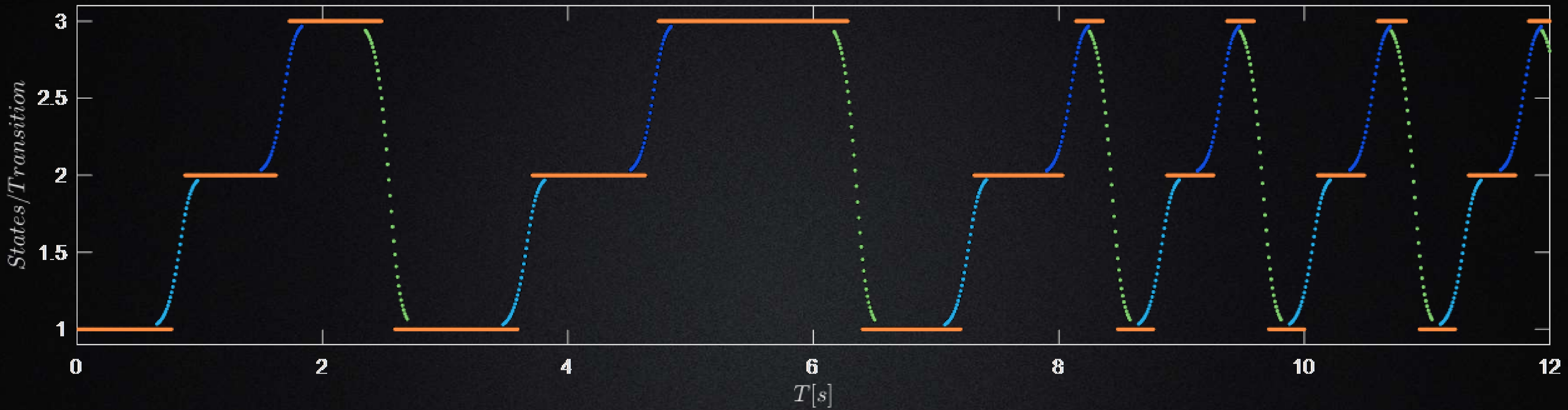
- It is essentially a state-phase-system
- All transitions are also reversible
- The state vector can be used to generate required profiles, which are state depended.



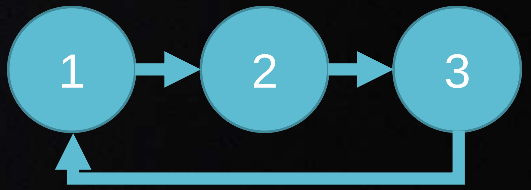
Attractor shape for a three-state cycle



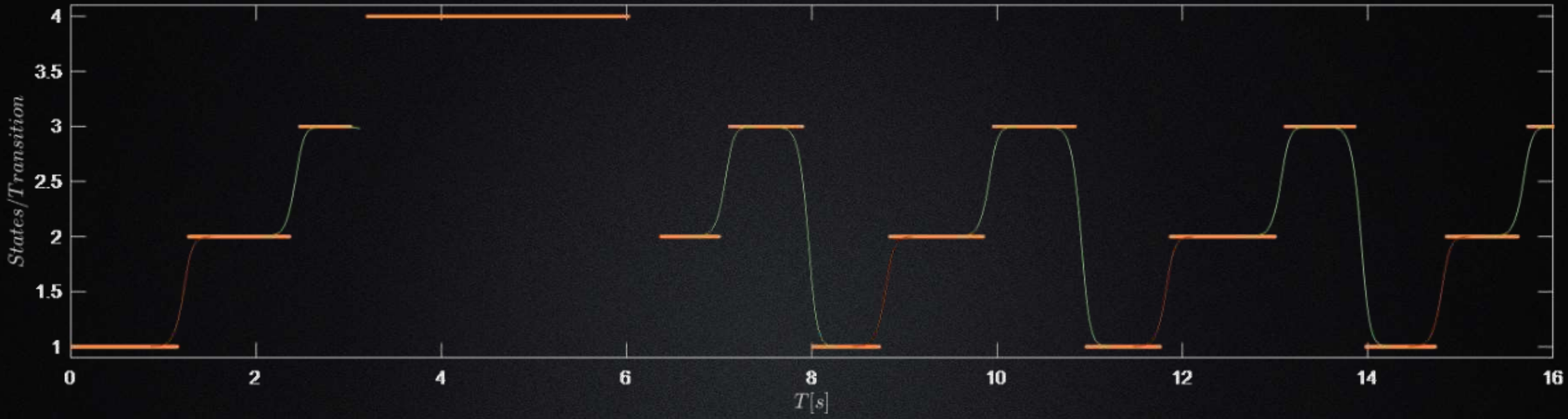
Example of stopping and leaving



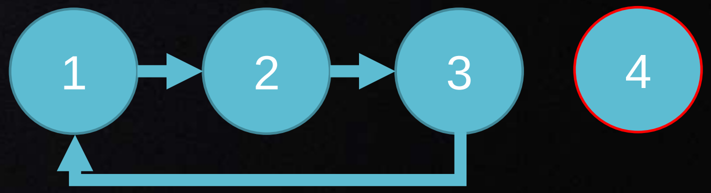
- Adding a negative input bias causes the system to stop at this state, adding a non-negative pulse at t=6s triggers the continuation of the sequence of states.



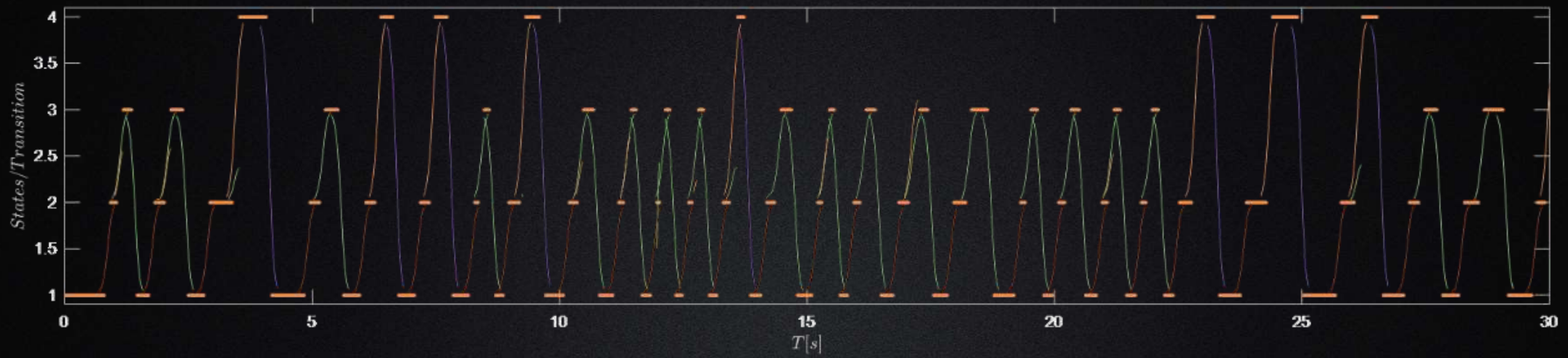
Example of excepting to error state



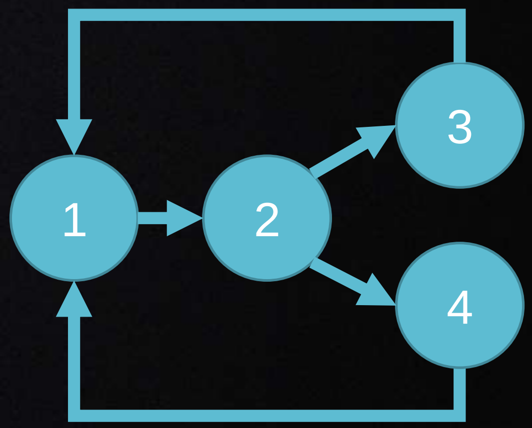
- We can still force the system to transition to any state at any time by applying a large enough pulse to bias input
- Useful to implement resets and to enter states that are not reachable during normal operation (i.e. error states)



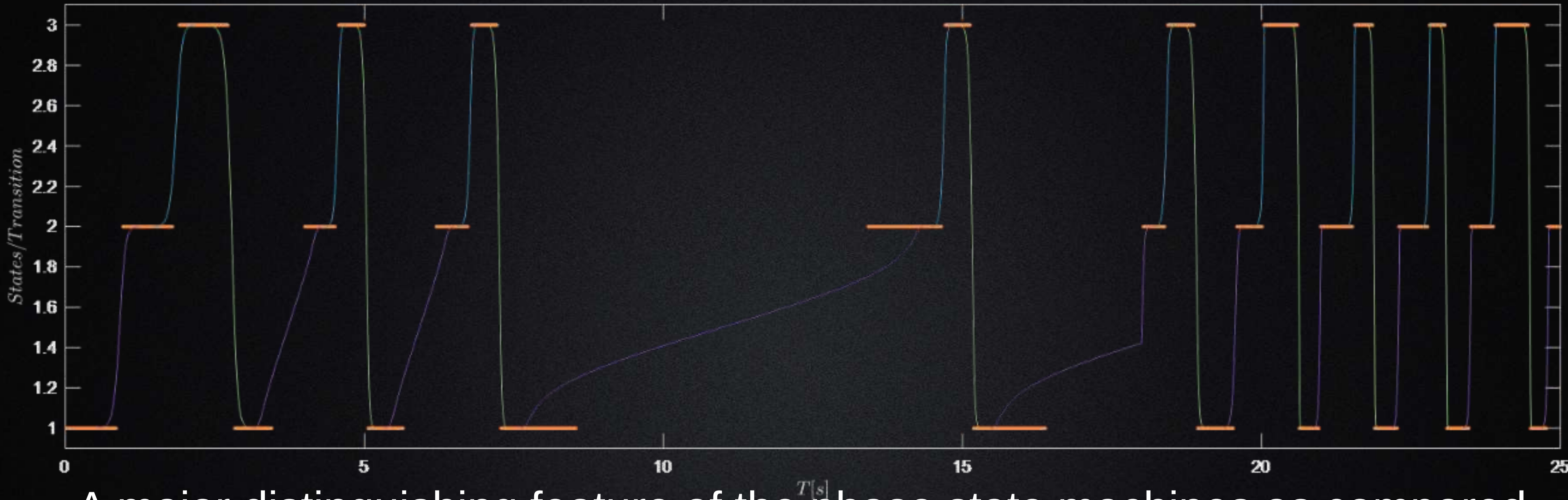
Probabilistic decisions



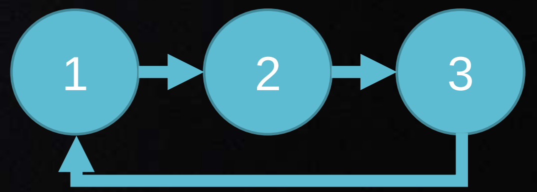
- Built-in ability to select transition probability.
- In this example, state 3 is visited 3 times more often than state 4.



Slowing Down and Speeding Up Transitions



- A major distinguishing feature of the phase state machines as compared to regular (discrete) state machines is the transitions of non-negligible duration.
- The transition periods (phase velocities) can be adjusted individually for each transition



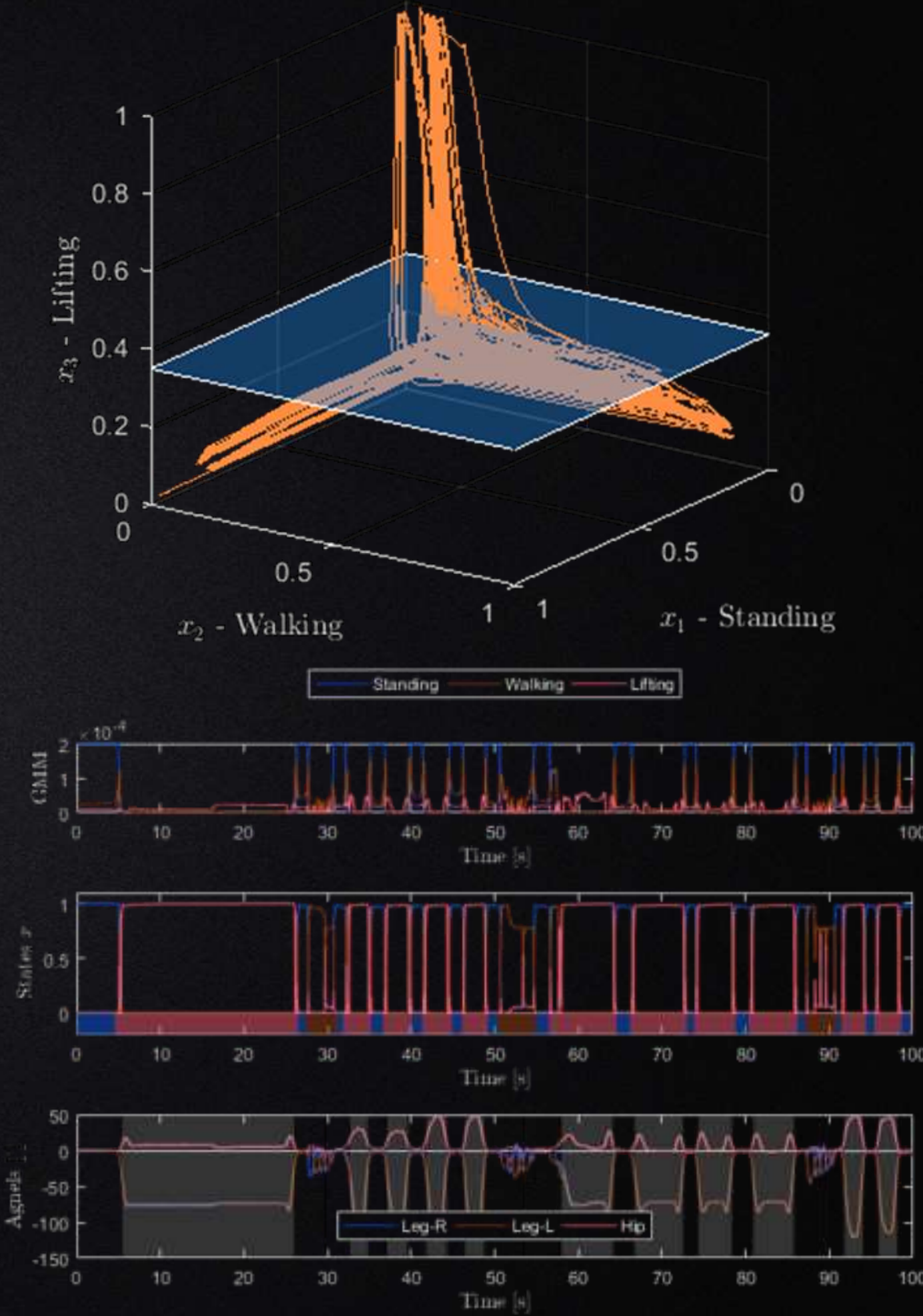
SHC and GMM for clasification

- Combined with Gaussian Mixture Models (GMM)

Human motion → GMM → SHC → States

- All transitions are also reversible

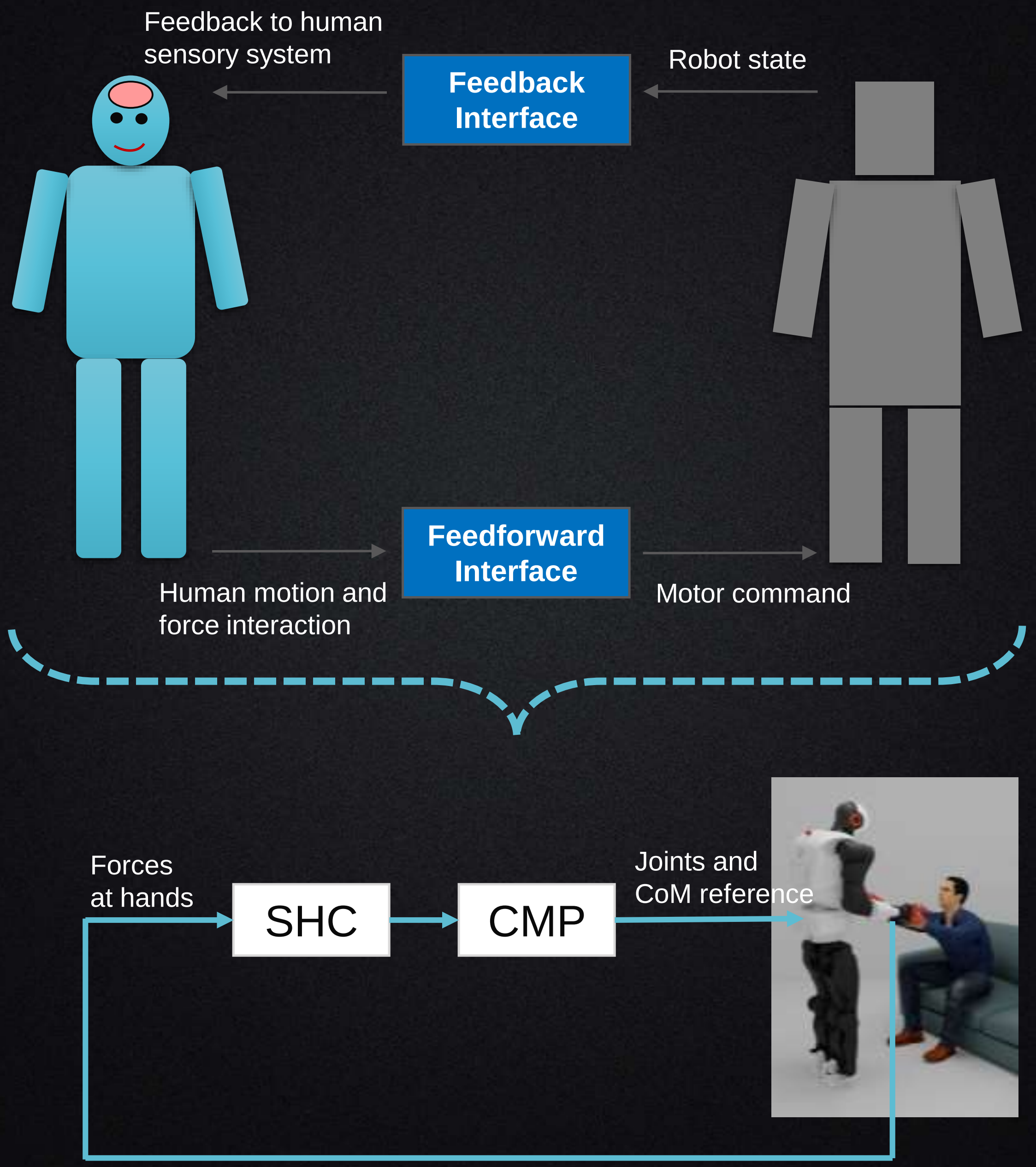
- The state vector can be used to generate required profiles, which are state depended.



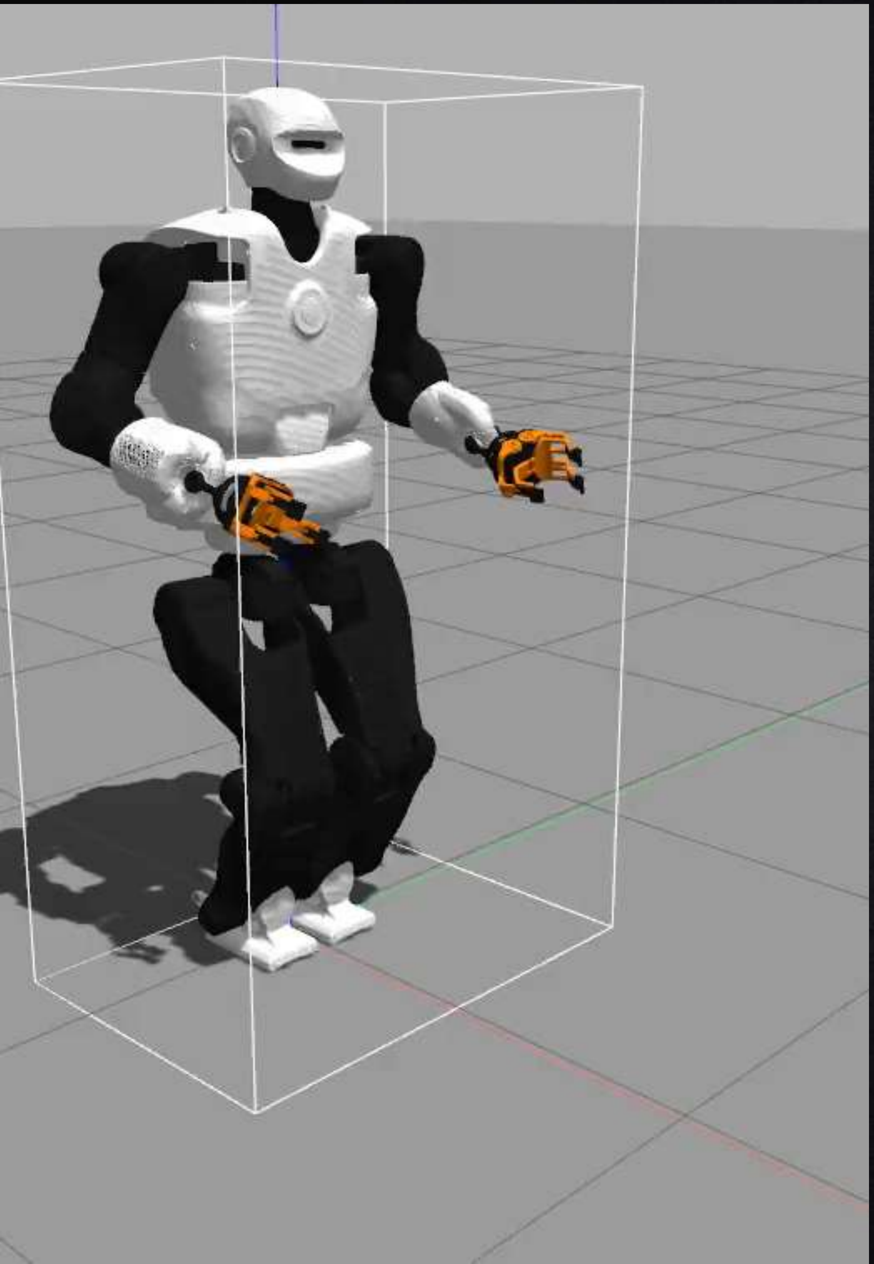
Petrič, T., et al. "Exoskeleton Control Based on Network of Stable Heteroclinic Channels (SHC) Combined with Gaussian Mixture Models (GMM)." International Symposium on Advances in Robot Kinematics. Springer, Cham, 2020.

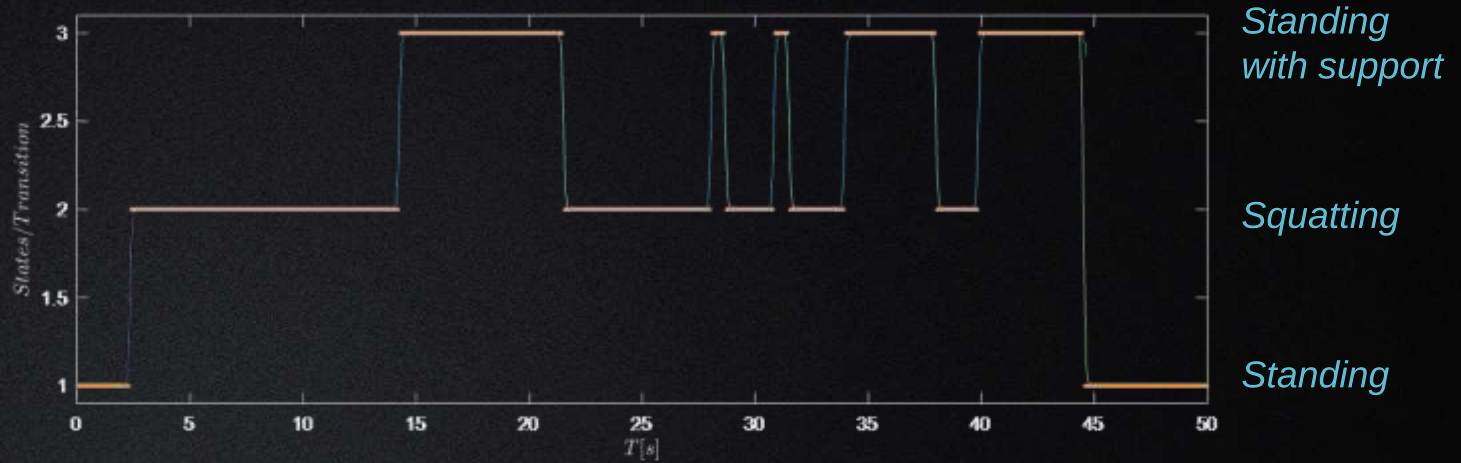
SHC for Continuous, Sequential and Reactive Behaviors

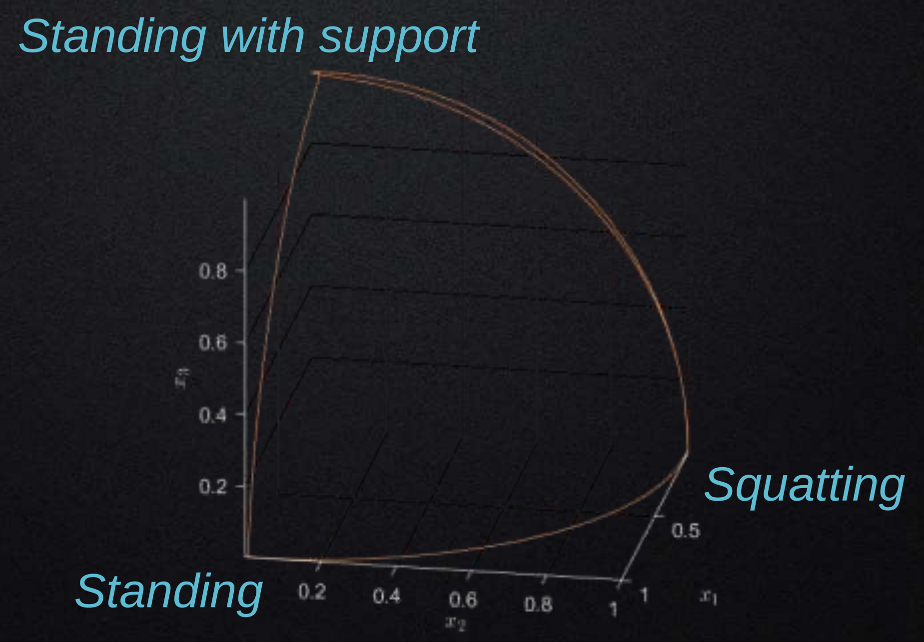
SHC controller

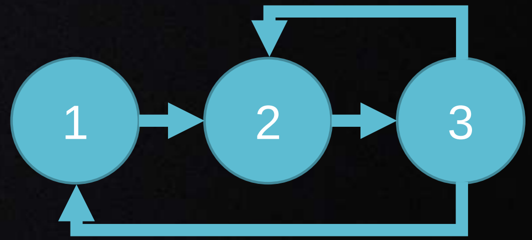


Simulation example









CoBoTaT

Advances in phase-state-systems

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